Algebra II
Common Midterm Review

Name

## Part I.

- I can use clear definitions, including correct mathematical language and notation.
- I can express inequalities in interval notation.
- I can determine if a relation is a function or not.
- I can determine if a function is discrete or continuous.

Express each inequality in interval notation.

1. $x<-4$ or $x \geq 3$
2. $-9 \leq d<0$ or $7>\mathrm{d} \geq 3$

Determine if each relation is a function. If it is not, please explain why. If it is a function,

- state the domain and range.
- state whether the function is discrete, continuous, or neither.

3. 


4.

| $x$ | $y$ |
| :---: | :---: |
| 3 | 10 |
| -5 | 10 |
| 4 | 9 |
| 7 | 5 |
| 12 | 4 |
| 0 | -6 |
| 2.8 | -12 |

5. 



## Part II.

- I can evaluate functions.
- I can perform operations on functions (addition, subtraction, multiplication, division).
- I can find values of $x$ that yield a given value of $y$.
- I can find zeroes and intercepts of a function given an equation.

Given the functions below, evaluate each expression.

$$
f(x)=-7 x+3 \quad g(x)=-x^{2}+4 x-2
$$

Graph of h(x)


1. Find $g(-4)$.
2. Find $(f \cdot g)(x)$.
3. Find $g(3 x-2)$.
4. Find $f(-5)-3[h(2)]^{2}$.
5. Find $(f g-h)(-1)$.
6. Find $k(1) \cdot[f(x)]^{k(-4)}-g(3 x)$.
7. Find all values of $g(x)$ such that $g(x)=-14$.
8. Find all zeroes of $g(x)$.
9. Find all values of $f(x)$ such that $f(x)=\frac{2}{3}$.
10. Find the x -and y -intercepts for $g(x)$.
11. TRUE OR FALSE: $\sqrt{25}= \pm 5$

## Part III.

- I can evaluate piecewise functions.
- I can graph piecewise functions.
- I can write an equation of a piecewise function.

Given the function $p(x)=\left\{\begin{array}{cc}2 & \text { when } x<-3 \\ x^{2}-2 x+4 & \text { when }-3 \leq x<2 \text {, answer each question. } \\ -5 x+4 \text { when } x>2\end{array}\right.$

1. Find $p(-5)$.
2. Find $p(-1)$.
3. Find $p(2)$.
4. Find $p(6)$.
5. Find $p\left(\frac{5}{4}\right)$.
6. Find all zeroes of $p(x)$.
7. Sketch the graph of $p(x)$.
8. Find the equation of the function whose graph is shown.


## Part IV.

- I can interpret graphs (domain, range, intervals of increase, decrease, constancy, zeroes, intercepts, relative and absolute extrema, intervals on which a function assumes particular values).
- I can graph a function using transformations of graphs of functions, including reflections and translations.
- I can find domain and range of a transformation of a function.
- I can sketch a graph given various conditions.

1-2. Complete the table for the functions whose graphs are shown.

|  | 1. | 2. |
| :---: | :---: | :---: |
| Domain |  |  |
| Range |  |  |
| Intervals of Increase |  |  |
| Intervals of Decrease |  |  |
| Constant Intervals |  |  |
| y-intercept |  |  |
| x-intercept(s) |  |  |
| Zeroes |  |  |
| Interval(s) on Which $\mathrm{f}(\mathrm{x})<-2$ |  |  |
| Interval(s) on Which $f(x)$ is Positive |  |  |
| $\mathrm{f}(1)=$ |  |  |
| Absolute Minimum |  |  |
| Absolute Maximum |  |  |
| Relative Minima |  |  |
| Relative Maxima |  |  |

## 3-5. Use knowledge of parent functions to graph each function.

3. $y=-|x-4|+3$
4. $y=2^{-x}-1$
5. $y=(-3-x)^{3}+2$

6-7. Given the graph of $g(x)$, graph each transformation.
Graph of $g(x)$
6. $y=-g(x+3)-2$
7. $y=g(2-x)+1$

8. Sketch a graph of a function satisfying the given conditions.

- Range: $y \in(-\infty, 7]$
- Zeros: $x=-4,-1,3$
- y-intercept: $(0,5)$
- Relative maximum: $f(x)=5$
- Relative minimum : $f(x)=-4$
- $f(5)$ is undefined
- $f(-2)=-1$

9. TRUE OR FALSE: If a function f is decreasing on an interval $(\mathrm{a}, \mathrm{b})$, then $f(a)<f(b)$.

## Part V.

- I can factor polynomials using common factors, regrouping, and special products.

Factor each polynomial completely.

1. $2 x^{3}+x^{2} \quad 18 x \quad 9$
2. $12 x^{2} x+6$
3. $16 x^{4}+54 x$
4. $x^{2}-13 x-48$
5. $2 x^{2}+17 x+21$
6. $9 p^{2} r+73 p r+70 r$
7. $(3 x+1)\left(\begin{array}{ll}x & 3\end{array}\right)+\left(\begin{array}{ll}x & 1\end{array}\right)\left(\begin{array}{ll}3 & x\end{array}\right)$
8. $8 m c \quad 24 m k+5 n c 15 n k$
9. $(2 x+3)^{2}-49$
10. $50 a^{2}+288$

## Part VI.

- I can solve quadratic equations using three methods (extracting roots, factoring, Quadratic Formula) with both real and non-real solutions.
- I can find discriminants and apply them to determine the number and nature of solutions to a quadratic equation.
- I can solve systems of quadratic equations.
- I can solve quadratic inequalities.
- I can solve applications of quadratics.


## Solve each equation by extracting roots.

1. $3(x+3)^{2}-10=-7$
2. $(d-3)^{2}+7=2$

## Solve each equation using the Quadratic Formula.

3. $2 x(x-4)+12=15$
4. $9 c^{2}-30 c=-25$

## Solve each equation using the Zero Product Property.

5. $x^{2}=9 x$
6. $7 a^{2}-12=-25 a$
7. Solve the system of equations.

$$
\begin{aligned}
& y=2 x^{2}+3 x \\
& y=x^{2}+8 x-4
\end{aligned}
$$

Solve each quadratic inequality. Write your final answers in interval notation.
8. $-3(x+2)^{2} \leq-12$
9. $x^{2}+4 x-8>0$
10. Use the fact that $2\left(\begin{array}{ll}2 x & 3\end{array}\right)^{3}+5=59$ to solve $2\left(\begin{array}{ll}2 x & 3\end{array}\right)^{3}+5=20 x \quad 1$.
11. Fill in the table below.

| Equation | Discriminant | Number and <br> Nature <br> of Solutions | What are the <br> SOLUTIONS <br> of the <br> equation? | What are the <br> x-intercepts <br> of the graph? |
| :--- | :--- | :--- | :--- | :--- |
| $x^{2}-2 x-15=0$ |  |  |  |  |
| $x^{2}-2 x+5=0$ |  |  |  |  |
| $-5 x^{2}=-75$ |  |  |  |  |
| $9 x^{2}-30 x+25=0$ |  |  |  |  |

12. The function $h(t)=16 t^{2} \quad 36 t+21$ represents the height (in feet) of a rocket $t$ seconds after being fired. What is the initial height of the rocket?
13. Peter throws a ball upward. The height of the ball, in feet, above the ground can be approximated by the function $h=-5 t^{2}+160 t$, where $t$ represents the amount of time, in seconds, since the ball has been released. In how many seconds wilt $h$ ball hit the ground?
14. The owner of a company that produces handcrafted music stands hires a consultant to help set the selling price for the product. The consultant analyzes the production costs and the consumer demand for the stands and arrives at a function for the profit, $P(x)=-0.3 x^{2}+75 x-2000$, where $x$ represents the selling price of the stands.
a. At what price should the stands be sold to earn the maximum profit that the company can make?
b. According to the function given, what is the maximum profit that the company can make?
c. What are the break-even points (the selling prices such that the profit is 0 )?

Give your answer to the nearest cent.
d. For which values of $x$ does the company make a profit?
e. For which values of $x$ does the company suffer a loss?

## Part VII.

- I can use the equation of a quadratic function in any form to find domain, range, concavity, points on a graph, zeroes, roots, x -intercept(s), y-intercept, vertex, and axis of symmetry.
- I can graph quadratic functions and inequalities.
- I can convert between forms of quadratic equations.
- I can find equations and inequalities of quadratics from a corresponding graph.

1. Convert the equation to vertex form. $y=2 x^{2}-5 x+3$
2. Convert the equation to standard form. $y=-\frac{1}{2}(x+4)(x-5)$
3. Convert the equation to intercept form. $y=3(x-4)^{2}-27$

Complete each prompt and graph the functions.

|  | 4. | 5. | 6. |
| :---: | :---: | :---: | :---: |
|  | $y=2\left(\begin{array}{ll}x & 4\end{array}\right)^{2} \quad 16$ | $y=-\frac{2}{3}(x+4)(x-2)$ | $y=x^{2}+3 x+4$ |
| Concavity |  |  |  |
| Vertex |  |  |  |
| Axis of Symmetry |  |  |  |
| Domain |  |  |  |
| Range |  |  |  |
| Zeroes |  |  |  |
| x-intercept(s) |  |  |  |
| y-intercept |  |  |  |
| Two Other Points |  |  |  |

7. Use your work from Question \#6 to graph $y>-x^{2}-3 x-4$.

Find the equation or inequality of the function whose graph is shown.
8.

9.

10. Find an equation of a quadratic function such that the graph has a vertex at $(3,2)$ and contains the point $(0,4)$.
11. TRUE OR FALSE: The graph of the function $y=2(x-3)^{2}-4$ has an absolute minimum.

## Part VIII.

- I can find square roots of negative numbers.
- I can add, subtract, multiply, and divide imaginary numbers.
- I can evaluate powers of $i$.

Simplify each expression completely.

1. $\sqrt{-96}$
2. $\sqrt{-12} \cdot \sqrt{-6}$
3. $4 \sqrt{-72}-\sqrt{-50}$
4. $\frac{7 i}{2 i^{2}}$
5. $i^{295}$
6. $\sqrt{-5} \cdot \sqrt{-4}+6 \sqrt{-45}$
7. $i^{-13}$
8. $\frac{3}{i}$

## Part IX.

- I can find opposites and conjugates of complex numbers.
- I can add, subtract, multiply, and divide complex numbers.
- I can find the modulus (absolute value) of complex numbers.
- I can solve equations equating complex numbers.


## Simplify each expression completely.

1. $(7+5 i)-(3-4 i)$
2. $(1+5 i)(7-3 i)$
3. $i(5-3 i)-(-4+5 i)$
4. $(5-3 i)(-4+5 i)$
5. $\frac{5 \quad 2 i}{10}$
6. $(-1-3 i)^{3}$
7. $\frac{4+i}{3 i}$
8. $\frac{3 \quad 2 i}{53 i}$
9. Find and simplify the reciprocal of $-4+5 \mathrm{i}$.
10. Find the absolute value of $-4+5$ i.

Find the opposite and conjugate for complex number.
11. $12-5 \mathrm{i}$
12. -5 i
13. 35
14. Find the values of $a$ and $b$ that make the equation true.

$$
(3 a-2 b)+(a+5 b) i=14-i
$$

